A rotating gravitational ellipse

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Abstract

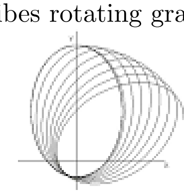
A gravitational ellipse is the mathematical result of Newton's law of gravitation. [Ref.1] The equation describing such an ellipse, is obtained by differentiating space-by-time twice. Le Verrier [Ref.2] stated: 'rotating gravitational ellipses are observed in the solar system'. One could be asked, to adjust the existing gravitational equation in such a way, that a rotating gravitational ellipse is obtained. The additional rotation is an extra variable, so the equation will be a three times space-by-time differentiated equation. In order to obtain a three times space-by-time differentiated equation we need to differentiate space-by-time for the third time. Differentiating space-by-time twice gives the following result.[Ref.3]

$$(\ddot{X})^2 + (\ddot{Y})^2 = (\ddot{R} - R\dot{a}^2)^2 + (R\ddot{a} + 2\dot{R}\dot{a})^2 \tag{1}$$

A third time differentiation of space-by-time gives the result:

$$(\dddot{X})^2 + (\dddot{Y})^2 = (\dddot{R} - 3\dot{R}\dot{a}^2 - 3R\dot{a}\ddot{a})^2 + (R\dddot{a} + 3\dot{R}\ddot{a} + 3\ddot{R}\dot{a} - R\dot{a}^3)^2 \tag{2}$$

We are now simply performing the necessary mathematical exercise to produce the new equation, which describes rotating gravitational ellipses.



I assume that the reader accepts the mathematical differential equation, which defines a rotating gravitational motion as observed. But we now have two equations defining rotating gravitational ellipses as observed in nature: the EIH equations (Ref.4) and the above equation 2, which obeys the Euclidean space premises.

The differentiation of space by time

In the Euclidean space we have X = R cos(a) and Y = R sin(a). Squaring X and squaring Y and then adding them results in:

$$(X)^2 + (Y)^2 = (R)^2 \tag{3}$$

as $cos^2(a)+sin^2(a) = 1$. The differentiation by time of X = R cos(a) and Y = R sin(a) results in: $(\dot{X}) = (\dot{R})cos(a) - R\dot{a}sin(a)$ and $(\dot{Y}) = (\dot{R})sin(a) + R\dot{a}cos(a)$ Squaring these last two equations and adding them results in:

$$(\dot{X})^2 + (\dot{Y})^2 = (\dot{R})^2 + (R\dot{a})^2 \tag{4}$$

Coriolis has differentiated space-by-time twice so repeat this procedure and equation 1 is the result. The result of the third space-by-time differentiation is equation 2 and the full derivation is in the appendix.

We are used in reasoning with forces and we make pictures of forces explaining the situations. If we multiply equation 1 with mass m then; The force in the x-direction Fx is $m\ddot{X}$. The force in the y-direction Fy is $m\ddot{Y}$. The force in the radial direction consists of two components: $m\ddot{R}$ and $-1mR\dot{a}^2$. The force in the angular direction consists of two components: $mR\ddot{a}$ and $+2m\dot{R}\dot{a}$. The last component is called the Coriolis force. The Coriolis force was unknown and unseen until the second differentiation of space by time. The third time differentiation of space by time results in new unseen mathematical terms just like the second time differentiation of space by time. In the radial direction we see: $-3\dot{R}\dot{a}^2$ and $-3R\dot{a}\ddot{a}$. In the angular direction we see: $+3\dot{R}\ddot{a}$ and $+3\ddot{R}\dot{a}$ and $-R\dot{a}^3$. Differentiating a force in the radial direction Fr or a force in the angular direction Fa will not lead to these terms. The mathematical terms of the third time differentiation of space by time are unique and unrelated. The mathematics created these terms. Differentiating a force in the radial direction or differentiation of a force in the angular direction does not lead to these new terms. Reasoning in words will not lead to these new mathematical terms. If a mathematical problem includes the third time differentiation of space by time equation 2 should be used. The differentiation of energy by time is $\dot{E} = dE/dt$. Energy is a force over a distance. $E = Fdx$. This results in $dE/dt = d(Fdx)/dt = (dF/dt)dx + Fdx/dt = (\dot{F})dx + F\dot{x} = m(\dddot{x}dx) + F\dot{x}$. From equations 1 and 2 we know that there is a difference between a force being differentiated by time and a third order interaction. There is a difference between $d(F)/dt = (\dot{F})$ and $m\dddot{x}$ This non-equality is important in order to do the correct mathematics. The term $m\dddot{x}$ is created through multiplying equation 2 with mass m. The term $d(F)/dt = (\dot{F})$ is created by multiplying with m and differentiating by time of equation 1. The two-dimensional nature and differentiating space by time created this unforeseen complexity.

Third-order equations

Trajectories of planets are described using the following two equations.

$$\ddot{R} - R\dot{a}^2 - C/R^2 = 0 \tag{5}$$

$$R\ddot{a} + 2\dot{R}\dot{a} = 0 \tag{6}$$

Differentiate these equations 5 and 6 by time and one obtains:

$$\dddot{R} - \dot{R}\dot{a}^2 - 2R\dot{a}\ddot{a} + 2C\dot{R}/R^3 = 0 \tag{7}$$

$$R\dddot{a} + \dot{R}\ddot{a} + 2\ddot{R}\dot{a} + 2\dot{R}\ddot{a} = 0 \tag{8}$$

Rearrange the equations:

$$\dddot{R} = \dot{R}\dot{a}^2 + 2R\dot{a}\ddot{a} - 2C\dot{R}/R^3 \tag{9}$$

$$R\dddot{a} = -\dot{R}\ddot{a} - 2\ddot{R}\dot{a} - 2\dot{R}\ddot{a} \tag{10}$$

From the third order space by time relation , equation 2, we have:

$$\mathrm{Gr/m} = \dddot{R} - 3\dot{R}\dot{a}^2 - 3R\dot{a}\ddot{a} \tag{11}$$

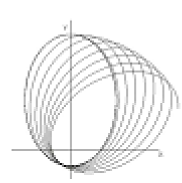
$$\mathrm{Ga/m} = R\dddot{a} + 3\dot{R}\ddot{a} + 3\ddot{R}\dot{a} - R\dot{a}^3 \tag{12}$$

As the symbol F is used for force, the symbol G is used for a third-order interaction. Replace the $\dddot{R}$ and the $\dddot{a}$ and one will get a new interaction.

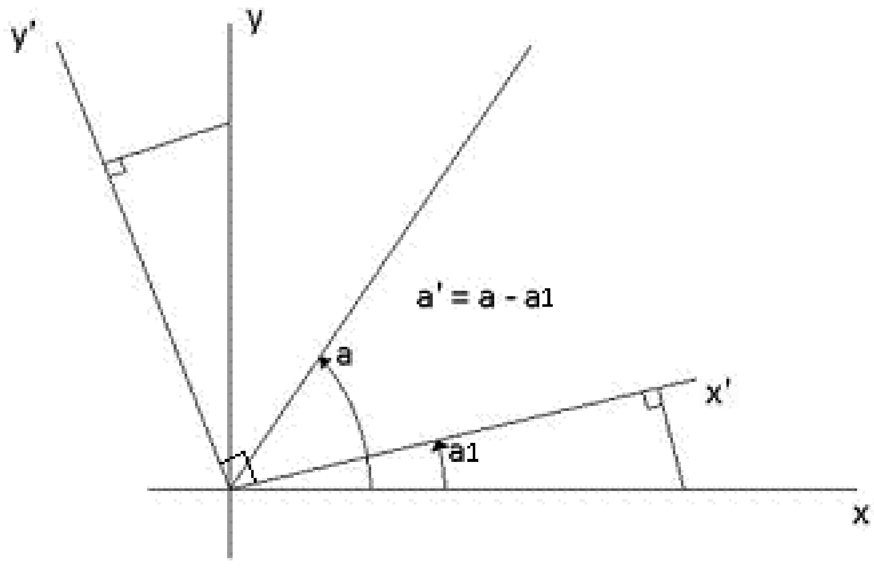
$$\mathrm{Gr/m} = -2\dot{R}\dot{a}^2 - R\dot{a}\ddot{a} - 2C\dot{R}/R^3 \tag{13}$$

$$\mathrm{Ga/m} = \ddot{R}\dot{a} - R\dot{a}^3 \tag{14}$$

We now have the new third order interaction and let the computer do the actual calculation.



Le Verrier stated that the gravitational ellipse should rotate. The equations 5 and 6 should be transformed rotationally. This transformation is like a velocity transformation in the x direction, but then in the angular direction.



We have X = Rcos(a) and Y=Rsin(a).

$$X' = cos(a1)X + sin(a1)Y \tag{15}$$

$$Y' = -1sin(a1)X + cos(a1)Y \tag{16}$$

-

$$R'cos(a1) = cos(a1)Rcos(a) + sin(a1)Rsin(a) \tag{17}$$

$$R'sin(a1) = -sin(a1)Rcos(a) + cos(a1)Rsin(a) \tag{18}$$

-

$$(R')^2 = (R)^2 \tag{19}$$

$$R' = R \tag{20}$$

-

$$cos(a') = cos(a1)cos(a) + sin(a1)sin(a) \tag{21}$$

$$sin(a') = -sin(a1)cos(a) + cos(a1)sin(a) \tag{22}$$

-

$$cos(a') = cos(a - a1) \tag{23}$$

$$sin(a') = sin(a - a1) \tag{24}$$

-

$$a' = a - a1 \tag{25}$$

$$a1 = wt \tag{26}$$

$$a' = a - wt \tag{27}$$

$$\dot{a}' = \dot{a} - w \tag{28}$$

$$\ddot{a}' = \ddot{a} \tag{29}$$

$$\dot{R}' = \dot{R} \tag{30}$$

$$\ddot{R}' = \ddot{R} \tag{31}$$

From Kepler ,we have:

$$\dot{a}R^2 = constant \tag{32}$$

So:

$$(\dot{a}' + w)R'^2 = constant \tag{33}$$

From now on we will leave out the quote for indication of the new system and recalculate the interaction equation. Differentiate the Kepler law and this will get the new Coriolis equation.

$$\ddot{a}R^2 + (\dot{a} + w)2R\dot{R} = 0 \tag{34}$$

Divide this equation by R.

$$\ddot{a}R + (\dot{a} + w)2\dot{R} = 0 \tag{35}$$

Differentiate this equation by time.

$$\dddot{a}R + \ddot{a}\dot{R} + 2\ddot{a}\dot{R} + 2\dot{a}\ddot{R} + 2w\ddot{R} = 0 \tag{36}$$

Rearrange the equation:

$$\dddot{a}R + 3\ddot{a}\dot{R} + 2(\dot{a} + w)\ddot{R} = 0 \tag{37}$$

Move the non-triple dot terms to the other side.

$$R\dddot{a} = -3\ddot{a}\dot{R} + -2(\dot{a} + w)\ddot{R} \tag{38}$$

The angular third order interaction is the following equation:

$$\mathrm{Ga/m} = R\dddot{a} + 3\dot{R}\ddot{a} + 3\ddot{R}\dot{a} - R\dot{a}^3 \tag{39}$$

Fill in the R angle triple dot from equation 38.

$$Ga/m = -3\ddot{a}\dot{R} + -2(\dot{a} + w)\ddot{R} + 3\dot{R}\ddot{a} + 3\ddot{R}\dot{a} - R\dot{a}^3 \tag{40}$$

Rearrange the equation:

$$Ga/m = (\dot{a} - 2w)\ddot{R} - R\dot{a}^3 \tag{41}$$

This equation is almost equal to equation 14. The difference is 2w and that is small. Mercury orbits the sun in 88 days and has a radius r of about 57 million km. The angular velocity is than 8e-7 rad/sec. The angular velocity out of the rosette motion is 35 km in one orbit. The angular velocity for the rosette motion is 8e-14 rad/sec. But one can use the exact equation.

Do the same for the radial interaction.

$$\ddot{R} - R\dot{a}^2 - C/R^2 = 0 \tag{42}$$

Fill in the new angular velocity.

$$\ddot{R}' - R'(\dot{a}' + w)^2 - C/R'^2 = 0 \tag{43}$$

Differentiate this by time and leave out the quote for the new system.

$$\dddot{R} = \dot{R}(\dot{a} + w)^2 + R2(\dot{a} + w)\ddot{a} - 2C\dot{R}/R^3 \tag{44}$$

The radial interaction of the third order was:

$$\mathrm{Gr/m} = \dddot{R} - 3\dot{R}\dot{a}^2 - 3R\dot{a}\ddot{a} \tag{45}$$

Replace the R triple dot. ($\dddot{R}$).

$$Gr/m = \dot{R}(\dot{a} + w)^2 + R2(\dot{a} + w)\ddot{a} - 2C\dot{R}/R^3 - 3\dot{R}\dot{a}^2 - 3R\dot{a}\ddot{a} \tag{46}$$

Write out all the terms and rearrange them.

$$Gr/m = -2\dot{R}\dot{a}^2 + 2w\dot{R}\dot{a} + \dot{R}w^2 + R(2w - \dot{a})\ddot{a} - 2C\dot{R}/R^3 \tag{47}$$

Rewrite the equation in the form to see the deviational terms.

$$Gr/m = -2\dot{R}\dot{a}^2 + R(2w - \dot{a})\ddot{a} - 2C\dot{R}/R^3 + 2w\dot{R}\dot{a} + \dot{R}w^2 \tag{48}$$

The first new interaction equation was:

$$\mathrm{Gr/m} = -2\dot{R}\dot{a}^2 - R\dot{a}\ddot{a} - 2C\dot{R}/R^3 \tag{49}$$

Term w is small compared to $\dot{a}$. So the equations are approximately equal. But one can always use the exact equation.

Conclusion

The differentiation of space by time as it was done by Coriolis revealed mathematical terms that were unknown and unseen before his mathematical exercise was performed. This mathematical exercise also reveals unknown and unseen mathematical terms. The two-dimensional nature of this exercise creates these mathematical terms. The important question is: Are these equations describing reality? Le Verrier states: 'Rotating gravitational ellipses are observed'. So, this mathematical exercise is just the workout of the accepted task of Le Verrier. The present mathematical workout of the mathematical exercise has been done in a Euclidean space. The result is therefore controversial. The EIH equations already exist for some time. Science should evaluate mathematical equations regarding their ability to mathematically describe reality. The mathematical equations derived in this exercise are meeting this requirement. Therefore these equations should be accepted by science.

The most important aspect of this mathematical exercise is the existence of the equations 1 and 2. Through these equations there are two ways of calculating $(\ddot{X}, \ddot{Y})$. The first way of calculating $(\ddot{X}, \ddot{Y})$ is through the differentiation by time of equation 1. The second way of calculating $(\ddot{X}, \ddot{Y})$ is through the direct differentiation of space by time. This result is equation 2. The problems and consequently accepted solutions of this issue need to be addressed first.

References

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2. Tisserand, M.F. (1880). 'Les Travaux de LeVerrier'. Annales de l'Observatoire de Paris, Memoires, XV (in French)., at SAO/NASA ADS

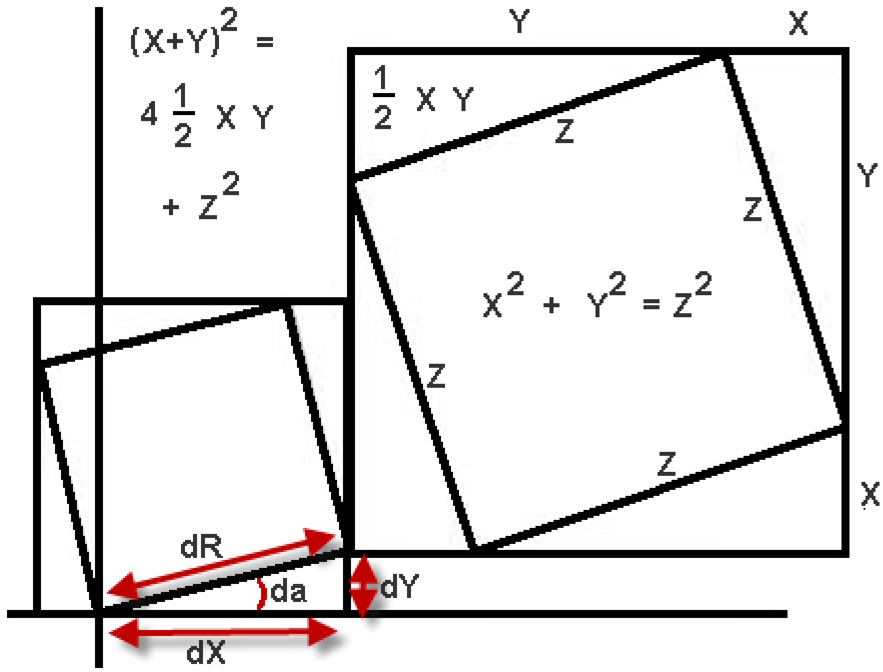
3. G-G Coriolis (1835). 'Sur les equations du mouvement relatif des systemes de corps'. J. De l'Ecole royale polytechnique 15: 144-154.

4. Einstein, A.; Infeld, L.; Hoffmann, B. (1938). "The Gravitational Equations and the Problem of Motion". Annals of Mathematics. Second series 39 (1): 65100. Bibcode:1938AnMat..39...65E. JSTOR 1968714.

`http://www.stefanboersen.nl/Contact/Coriolis_20120623_V10.htm`

Appendix

The first differentiation of space-by-time.



$$x = R\cos(a)$$
$$y = R\sin(a)$$

$$\dot{x} = \dot{R}\cos(a) - R\,\dot{a}\sin(a)$$
$$\dot{y} = \dot{R}\sin(a) + R\,\dot{a}\cos(a)$$

$$\dot{x}^2 + \dot{y}^2 = (\dot{R}\cos(a) - R\,\dot{a}\sin(a))^2 + (\dot{R}\sin(a) + R\,\dot{a}\cos(a))^2$$

$$\dot{x}^2 + \dot{y}^2 = \dot{R}^2\cos(a)^2 + R^2\dot{a}^2\sin(a)^2 - 2\,\dot{R}\cos(a)\,R\,\dot{a}\sin(a)$$
$$+\ \dot{R}^2\sin(a)^2 \quad R^2\dot{a}^2\cos(a)^2 + 2\,\dot{R}\sin(a)\,R\,\dot{a}\cos(a)$$

$$\dot{x}^2 + \dot{y}^2 = \dot{R}^2 + R^2\dot{a}^2$$

The second differentiation of space-by-time.

$$x = R\cos(a)$$
$$y = R\sin(a)$$

$$\begin{aligned}
\dot{x} &= \dot{R}\cos(a) - R\,\dot{a}\sin(a)\\
\dot{y} &= \dot{R}\sin(a) + R\,\dot{a}\cos(a)
\end{aligned}$$

$$\begin{aligned}
\ddot{x} &= \ddot{R}\cos(a) - \dot{R}\,\dot{a}\sin(a) - \dot{R}\,\dot{a}\sin(a) - R\,\dot{a}^2\cos(a) - R\,\ddot{a}\sin(a)\\
\ddot{y} &= \ddot{R}\sin(a) + \dot{R}\,\dot{a}\cos(a) + \dot{R}\,\dot{a}\cos(a) - R\,\dot{a}^2\sin(a) + R\,\ddot{a}\cos(a)
\end{aligned}$$

$$\begin{aligned}
\ddot{x} &= \ddot{R}\cos(a) - 2\,\dot{R}\,\dot{a}\sin(a) - R\,\dot{a}^2\cos(a) - R\,\ddot{a}\sin(a)\\
\ddot{y} &= \ddot{R}\sin(a) + 2\,\dot{R}\,\dot{a}\cos(a) - R\,\dot{a}^2\sin(a) + R\,\ddot{a}\cos(a)
\end{aligned}$$

$$\begin{aligned}
\ddot{x}^2 = {} & \ddot{R}^2\cos(a)^2 + 4\,\dot{R}^2\dot{a}^2\sin(a)^2 + R^2\dot{a}^4\cos(a)^2 + R^2\ddot{a}^2\sin(a)^2\\
& - 4\,\ddot{R}\cos(a)\,\dot{R}\,\dot{a}\sin(a) - 2\,\ddot{R}\cos(a)\,R\,\dot{a}^2\cos(a) - 2\,\ddot{R}\cos(a)\,R\,\ddot{a}\sin(a)\\
& + 4\,\dot{R}\,\dot{a}\sin(a)\,R\,\dot{a}^2\cos(a) + 4\,\dot{R}\,\dot{a}\sin(a)\,R\,\ddot{a}\sin(a)\\
& + 2\,R\,\dot{a}^2\cos(a)\,R\,\ddot{a}\sin(a)
\end{aligned}$$

$$\begin{aligned}
\ddot{y}^2 = {} & \ddot{R}^2\sin(a)^2 + 4\,\dot{R}^2\dot{a}^2\cos(a)^2 + R^2\dot{a}^4\sin(a)^2 + R^2\ddot{a}^2\cos(a)^2\\
& + 4\,\ddot{R}\sin(a)\,\dot{R}\,\dot{a}\cos(a) - 2\,\ddot{R}\sin(a)\,R\,\dot{a}^2\sin(a) + 2\,\ddot{R}\sin(a)\,R\,\ddot{a}\cos(a)\\
& - 4\,\dot{R}\,\dot{a}\cos(a)\,R\,\dot{a}^2\sin(a) + 4\,\dot{R}\,\dot{a}\cos(a)\,R\,\ddot{a}\cos(a)\\
& - 2\,R\,\dot{a}^2\sin(a)\,R\,\ddot{a}\cos(a)
\end{aligned}$$

$$\ddot{x}^2 + \ddot{y}^2 = \ddot{R}^2 + 4\,\dot{R}^2\dot{a}^2 + R^2\dot{a}^4 + R^2\ddot{a}^2 - 2\,\ddot{R}\,R\,\dot{a}^2 + 4\,\dot{R}\,\dot{a}\,R\,\ddot{a}$$

$$\ddot{x}^2 + \ddot{y}^2 = (\ddot{R} - R\,\dot{a}^2)^2 + (R\,\ddot{a} + 2\,\dot{R}\,\dot{a})^2$$

The third differentiation of space-by-time.

$$x = R\cos(a)$$
$$y = R\sin(a)$$

$$\dot{x} = \dot{R}\cos(a) - R\,\dot{a}\sin(a)$$
$$\dot{y} = \dot{R}\sin(a) + R\,\dot{a}\cos(a)$$

$$\ddot{x} = \ddot{R}\cos(a) - \dot{R}\,\dot{a}\sin(a) - \dot{R}\,\dot{a}\sin(a) - R\,\dot{a}^2\cos(a) - R\,\ddot{a}\sin(a)$$
$$\ddot{y} = \ddot{R}\sin(a) + \dot{R}\,\dot{a}\cos(a) + \dot{R}\,\dot{a}\cos(a) - R\,\dot{a}^2\sin(a) + R\,\ddot{a}\cos(a)$$

$$\ddot{x} = \ddot{R}\cos(a) - 2\,\dot{R}\,\dot{a}\sin(a) - R\,\dot{a}^2\cos(a) - R\,\ddot{a}\sin(a)$$
$$\ddot{y} = \ddot{R}\sin(a) + 2\,\dot{R}\,\dot{a}\cos(a) - R\,\dot{a}^2\sin(a) + R\,\ddot{a}\cos(a)$$

$$\begin{aligned}\dddot{x} = {} & \dddot{R}\cos(a) - 2\,\ddot{R}\,\dot{a}\sin(a) - \dot{R}\,\dot{a}^2\cos(a) - \dot{R}\,\ddot{a}\sin(a) \\ & - \ddot{R}\,\dot{a}\sin(a) - 2\,\dot{R}\,\ddot{a}\sin(a) - 2\,R\,\dot{a}\ddot{a}\cos(a) - R\,\dddot{a}\sin(a) \\ & - 2\,\dot{R}\,\dot{a}^2\cos(a) + R\,\dot{a}^3\sin(a) - R\,\ddot{a}\,\dot{a}\cos(a)\end{aligned}$$

$$\begin{aligned}\dddot{y} = {} & \dddot{R}\sin(a) + 2\,\ddot{R}\,\dot{a}\cos(a) - \dot{R}\,\dot{a}^2\sin(a) + \dot{R}\,\ddot{a}\cos(a) \\ & + \ddot{R}\,\dot{a}\cos(a) + 2\,\dot{R}\,\ddot{a}\cos(a) - 2\,R\,\dot{a}\ddot{a}\sin(a) + R\,\dddot{a}\cos(a) \\ & - 2\,\dot{R}\,\dot{a}^2\sin(a) - R\,\dot{a}^3\cos(a) - R\,\ddot{a}\,\dot{a}\sin(a)\end{aligned}$$

$$\dddot{X} = \dddot{R}\cos(a) - 3\ddot{R}\dot{a}\sin(a) - 3\dot{R}\dot{a}^2\cos(a) - 3\dot{R}\ddot{a}\sin(a) - 3R\dot{a}\ddot{a}\cos(a) + R\dot{a}^3\sin(a) - R\dddot{a}\sin(a)$$

$$\dddot{X}^2 = \dddot{R}^2\cos(a)^2 + 9\ddot{R}^2\dot{a}^2\sin(a)^2 + 9\dot{R}^2\dot{a}^4\cos(a)^2 + 9\dot{R}^2\ddot{a}^2\sin(a)^2 + 9R^2\dot{a}^2\ddot{a}^2\cos(a)^2 + R^2\dot{a}^6\sin(a)^2 + R^2\dddot{a}^2\sin(a)^2$$

$$- 6\dddot{R}\cos(a)\ddot{R}\dot{a}\sin(a) - 6\dddot{R}\cos(a)\dot{R}\dot{a}^2\cos(a) - 6\dddot{R}\cos(a)\dot{R}\ddot{a}\sin(a) - 6\dddot{R}\cos(a)R\dot{a}\ddot{a}\cos(a)$$

$$+ 2\dddot{R}\cos(a)R\dot{a}^3\sin(a) - 2\dddot{R}\cos(a)R\dddot{a}\sin(a)$$

$$+ 18\ddot{R}\dot{a}\sin(a)\dot{R}\dot{a}^2\cos(a) + 18\ddot{R}\dot{a}\sin(a)\dot{R}\ddot{a}\sin(a) + 18\ddot{R}\dot{a}\sin(a)R\dot{a}\ddot{a}\cos(a) - 6\ddot{R}\dot{a}\sin(a)R\dot{a}^3\sin(a)$$

$$+ 6\ddot{R}\dot{a}\sin(a)R\dddot{a}\sin(a)$$

$$+ 18\dot{R}\dot{a}^2\cos(a)\dot{R}\ddot{a}\sin(a) + 18\dot{R}\dot{a}^2\cos(a)R\dot{a}\ddot{a}\cos(a) - 6\dot{R}\dot{a}^2\cos(a)R\dot{a}^3\sin(a) + 6\dot{R}\dot{a}^2\cos(a)R\dddot{a}\sin(a)$$

$$+ 18\dot{R}\ddot{a}\sin(a)R\dot{a}\ddot{a}\cos(a) - 6\dot{R}\ddot{a}\sin(a)R\dot{a}^3\sin(a) + 6\dot{R}\ddot{a}\sin(a)R\dddot{a}\sin(a)$$

$$- 6R\dot{a}\ddot{a}\cos(a)R\dot{a}^3\sin(a) + 6R\dot{a}\ddot{a}\cos(a)R\dddot{a}\sin(a)$$

$$- 2R\dot{a}^3\sin(a)R\dddot{a}\sin(a)$$

$$\dddot{Y} = \dddot{R}\sin(a) + 3\ddot{R}\dot{a}\cos(a) - 3\dot{R}\dot{a}^2\sin(a) + 3\dot{R}\ddot{a}\cos(a) - 3R\dot{a}\ddot{a}\sin(a) - R\dot{a}^3\cos(a) + R\dddot{a}\cos(a)$$

$$\dddot{Y}^2 = \dddot{R}^2\sin(a)^2 + 9\ddot{R}^2\dot{a}^2\cos(a)^2 + 9\dot{R}^2\dot{a}^4\sin(a)^2 + 9\dot{R}^2\ddot{a}^2\cos(a)^2 + 9R^2\dot{a}^2\ddot{a}^2\sin(a)^2 + R^2\dot{a}^6\cos(a)^2 + R^2\dddot{a}^2\cos(a)^2$$

$$+ 6\dddot{R}\sin(a)\ddot{R}\dot{a}\cos(a) - 6\dddot{R}\sin(a)\dot{R}\dot{a}^2\sin(a) + 6\dddot{R}\sin(a)\dot{R}\ddot{a}\cos(a) - 6\dddot{R}\sin(a)R\dot{a}\ddot{a}\sin(a)$$

$$- 2\dddot{R}\sin(a)R\dot{a}^3\cos(a) + 2\dddot{R}\sin(a)R\dddot{a}\cos(a)$$

$$- 18\ddot{R}\dot{a}\cos(a)\dot{R}\dot{a}^2\sin(a) + 18\ddot{R}\dot{a}\cos(a)\dot{R}\ddot{a}\cos(a) - 18\ddot{R}\dot{a}\cos(a)R\dot{a}\ddot{a}\sin(a) - 6\ddot{R}\dot{a}\cos(a)R\dot{a}^3\cos(a)$$

$$+ 6\ddot{R}\dot{a}\cos(a)R\dddot{a}\cos(a)$$

$$- 18\dot{R}\dot{a}^2\sin(a)\dot{R}\ddot{a}\cos(a) + 18\dot{R}\dot{a}^2\sin(a)R\dot{a}\ddot{a}\sin(a) + 6\dot{R}\dot{a}^2\sin(a)R\dot{a}^3\cos(a) - 6\dot{R}\dot{a}^2\sin(a)R\dddot{a}\cos(a)$$

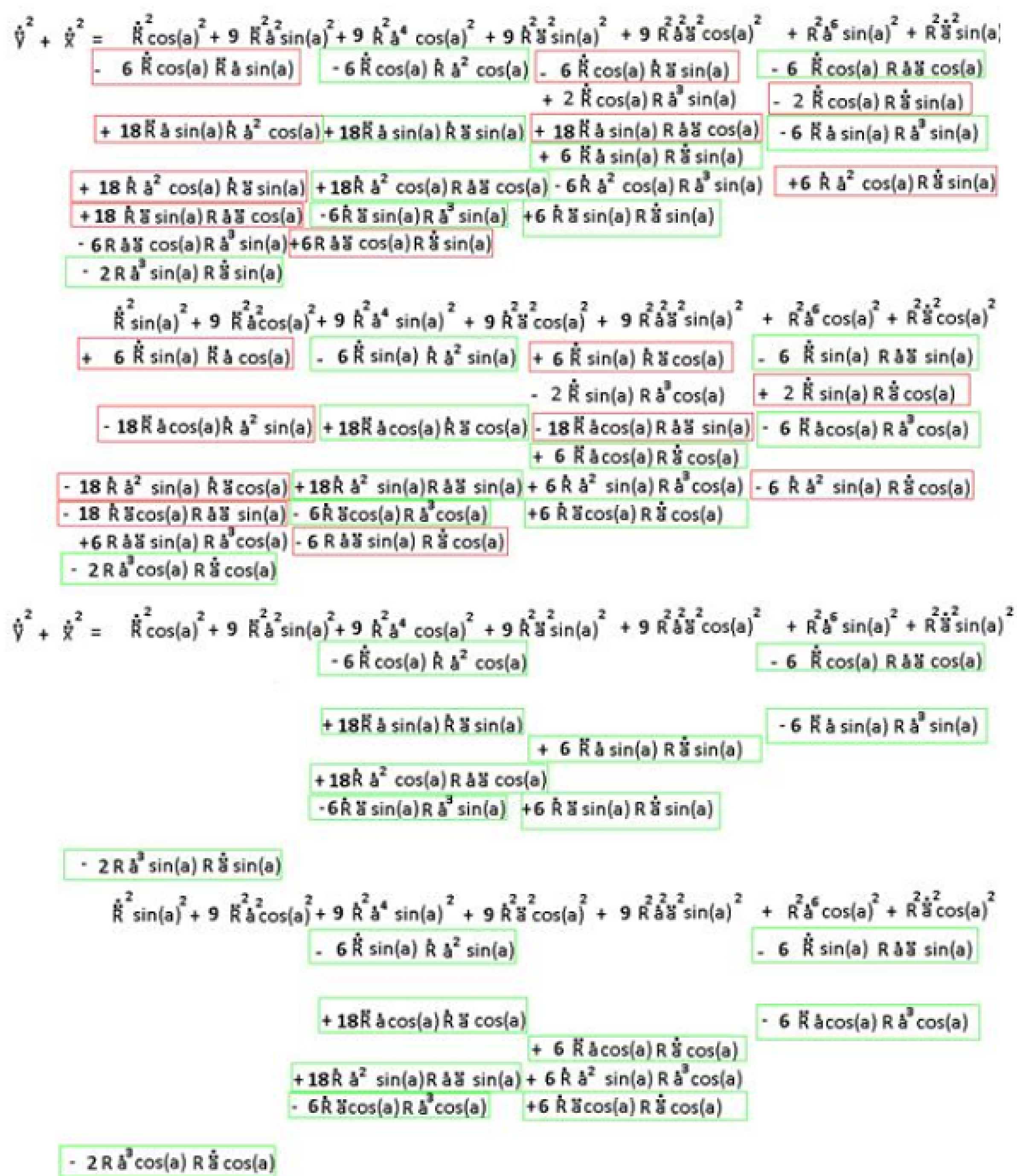
$$- 18\dot{R}\ddot{a}\cos(a)R\dot{a}\ddot{a}\sin(a) - 6\dot{R}\ddot{a}\cos(a)R\dot{a}^3\cos(a) + 6\dot{R}\ddot{a}\cos(a)R\dddot{a}\cos(a)$$

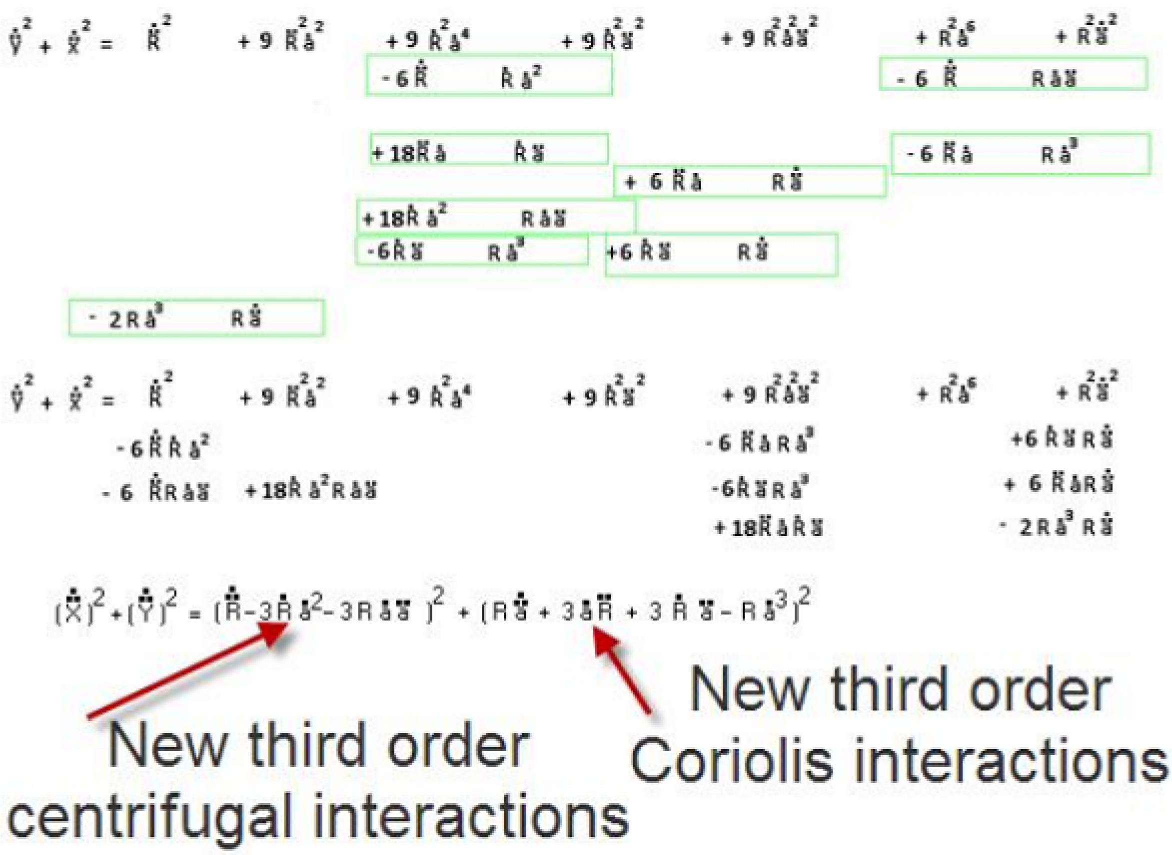
$$+ 6R\dot{a}\ddot{a}\sin(a)R\dot{a}^3\cos(a) - 6R\dot{a}\ddot{a}\sin(a)R\dddot{a}\cos(a)$$

$$- 2R\dot{a}^3\cos(a)R\dddot{a}\cos(a)$$

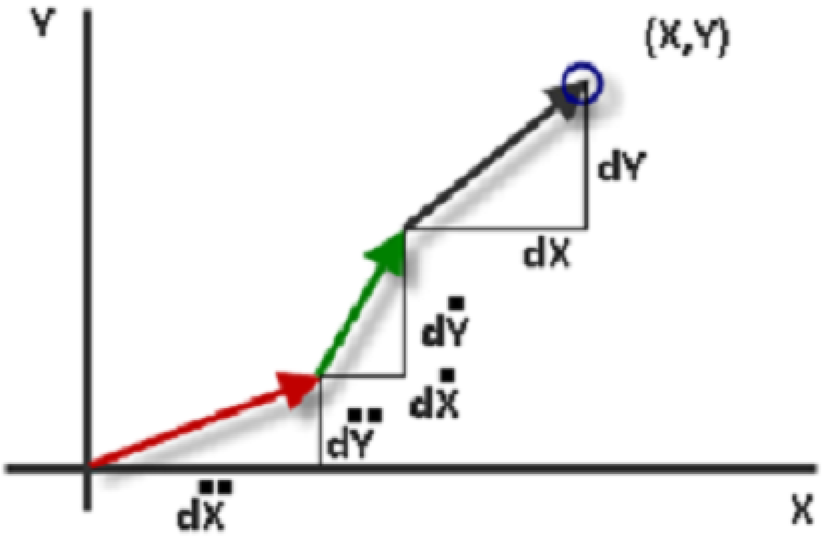
$$
\begin{aligned}
\dot{y}^2 + \dot{x}^2 = {} & \dddot{R}^2\cos(a)^2 + 9\,\ddot{R}^2\dot{a}^2\sin(a)^2 + 9\,\dot{R}^2\dot{a}^4\cos(a)^2 + 9\,\dot{R}^2\ddot{a}^2\sin(a)^2 + 9\,R^2\dot{a}^2\ddot{a}^2\cos(a)^2 + R^2\dot{a}^6\sin(a)^2 + R^2\dddot{a}^2\sin(a)^2 \\
& - 6\,\dddot{R}\cos(a)\,\ddot{R}\,\dot{a}\sin(a) - 6\,\dddot{R}\cos(a)\,\dot{R}\,\dot{a}^2\cos(a) - 6\,\dddot{R}\cos(a)\,\dot{R}\,\ddot{a}\sin(a) - 6\,\dddot{R}\cos(a)\,R\,\dot{a}\ddot{a}\cos(a) \\
& + 2\,\dddot{R}\cos(a)\,R\,\dot{a}^3\sin(a) - 2\,\dddot{R}\cos(a)\,R\,\dddot{a}\sin(a) \\
& + 18\,\ddot{R}\,\dot{a}\sin(a)\,\dot{R}\,\dot{a}^2\cos(a) + 18\,\ddot{R}\,\dot{a}\sin(a)\,\dot{R}\,\ddot{a}\sin(a) + 18\,\ddot{R}\,\dot{a}\sin(a)\,R\,\dot{a}\ddot{a}\cos(a) - 6\,\ddot{R}\,\dot{a}\sin(a)\,R\,\dot{a}^3\sin(a) \\
& + 6\,\ddot{R}\,\dot{a}\sin(a)\,R\,\dddot{a}\sin(a) \\
& + 18\,\dot{R}\,\dot{a}^2\cos(a)\,\dot{R}\,\ddot{a}\sin(a) + 18\,\dot{R}\,\dot{a}^2\cos(a)\,R\,\dot{a}\ddot{a}\cos(a) - 6\,\dot{R}\,\dot{a}^2\cos(a)\,R\,\dot{a}^3\sin(a) + 6\,\dot{R}\,\dot{a}^2\cos(a)\,R\,\dddot{a}\sin(a) \\
& + 18\,\dot{R}\,\ddot{a}\sin(a)\,R\,\dot{a}\ddot{a}\cos(a) - 6\,\dot{R}\,\ddot{a}\sin(a)\,R\,\dot{a}^3\sin(a) + 6\,\dot{R}\,\ddot{a}\sin(a)\,R\,\dddot{a}\sin(a) \\
& - 6\,R\,\dot{a}\ddot{a}\cos(a)\,R\,\dot{a}^3\sin(a) + 6\,R\,\dot{a}\ddot{a}\cos(a)\,R\,\dddot{a}\sin(a) \\
& - 2\,R\,\dot{a}^3\sin(a)\,R\,\dddot{a}\sin(a)
\end{aligned}
$$

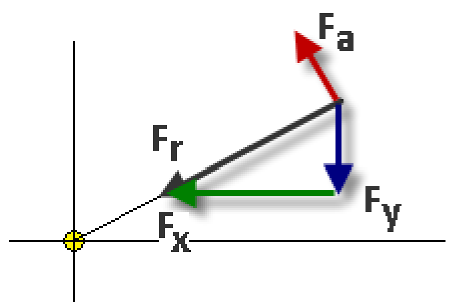
$$
\begin{aligned}
& \dddot{R}^2\sin(a)^2 + 9\,\ddot{R}^2\dot{a}^2\cos(a)^2 + 9\,\dot{R}^2\dot{a}^4\sin(a)^2 + 9\,\dot{R}^2\ddot{a}^2\cos(a)^2 + 9\,R^2\dot{a}^2\ddot{a}^2\sin(a)^2 + R^2\dot{a}^6\cos(a)^2 + R^2\dddot{a}^2\cos(a)^2 \\
& + 6\,\dddot{R}\sin(a)\,\ddot{R}\,\dot{a}\cos(a) - 6\,\dddot{R}\sin(a)\,\dot{R}\,\dot{a}^2\sin(a) + 6\,\dddot{R}\sin(a)\,\dot{R}\,\ddot{a}\cos(a) - 6\,\dddot{R}\sin(a)\,R\,\dot{a}\ddot{a}\sin(a) \\
& - 2\,\dddot{R}\sin(a)\,R\,\dot{a}^3\cos(a) + 2\,\dddot{R}\sin(a)\,R\,\dddot{a}\cos(a) \\
& - 18\,\ddot{R}\,\dot{a}\cos(a)\,\dot{R}\,\dot{a}^2\sin(a) + 18\,\ddot{R}\,\dot{a}\cos(a)\,\dot{R}\,\ddot{a}\cos(a) - 18\,\ddot{R}\,\dot{a}\cos(a)\,R\,\dot{a}\ddot{a}\sin(a) - 6\,\ddot{R}\,\dot{a}\cos(a)\,R\,\dot{a}^3\cos(a) \\
& + 6\,\ddot{R}\,\dot{a}\cos(a)\,R\,\dddot{a}\cos(a) \\
& - 18\,\dot{R}\,\dot{a}^2\sin(a)\,\dot{R}\,\ddot{a}\cos(a) + 18\,\dot{R}\,\dot{a}^2\sin(a)\,R\,\dot{a}\ddot{a}\sin(a) + 6\,\dot{R}\,\dot{a}^2\sin(a)\,R\,\dot{a}^3\cos(a) - 6\,\dot{R}\,\dot{a}^2\sin(a)\,R\,\dddot{a}\cos(a) \\
& - 18\,\dot{R}\,\ddot{a}\cos(a)\,R\,\dot{a}\ddot{a}\sin(a) - 6\,\dot{R}\,\ddot{a}\cos(a)\,R\,\dot{a}^3\cos(a) + 6\,\dot{R}\,\ddot{a}\cos(a)\,R\,\dddot{a}\cos(a) \\
& + 6\,R\,\dot{a}\ddot{a}\sin(a)\,R\,\dot{a}^3\cos(a) - 6\,R\,\dot{a}\ddot{a}\sin(a)\,R\,\dddot{a}\cos(a) \\
& - 2\,R\,\dot{a}^3\cos(a)\,R\,\dddot{a}\cos(a)
\end{aligned}
$$




New third order
centrifugal interactions
New third order
Coriolis interactions

Orthogonality of Euclidean space





Rotating gravitational ellipses